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Finite time thermodynamic analysis of endoreversible Stirling heat engine with regenerative losses

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Abstract

This communication presents an investigation of a finite time thermodynamic analysis of an endoreversible Stirling heat engine. Finite time thermodynamics has been applied to maximize the power output and the corresponding thermal efficiency of an endoreversible Stirling heat engine with internal heat loss in the regenerator and for the finite heat capacity of the external reservoirs. The effect of the effectiveness of the various heat exchangers, the inlet temperatures of external heat reservoirs on the power output and the corresponding thermal efficiency have been studied. It is seen that an endoreversible Stirling heat engine with an ideal regenerator ($\epsilon_R=1.00$) is as efficient as an endoreversible Carnot heat engine. It is also found that the maximum power output increases with the heat capacitance rates and effectiveness of the source/sink side heat exchangers while thermal efficiency increases with the effectiveness of the regenerator. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

Finite time thermodynamics/finite temperature difference thermodynamics deals with the fact that there must be a finite temperature difference between the working fluid/substance and the source/sink heat reservoirs (with which it is in contact) in order to transfer a finite amount of heat in a finite time. The literature of finite time thermodynamics began with the novel work of Curzon and Ahlborn [1], who established a theoretical model of a real Carnot heat engine at maximum power output with a different efficiency expression than the well-known Carnot efficiency. They assumed that due to the finite conductivity of materials, the heat engine is not operated between the available high and low temperatures (T_H and T_L) of source and sink but between the temperatures T_h and T_c of the hot and cold working fluids of the heat engine cycle,

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Nomenclature

A	area (m^2)
C	heat capacitance (kW/K)
n	numbers of moles
N	number of heat transfer units
P	power output (kW)
Q	heat (kJ)
T	temperature (K)
t	time (s)
U	overall heat transfer coefficient (W/K m^2)
V	volume of the working fluid (m^3)
W	output work (kJ)

Subscripts

c	cold side/sink side
h, H	hot side/heat source
L	heat sink
m	maximum
R	regenerator
$1, 2$	initial, final
$1, 2, 3, 4$	state points

Greek

η	thermal efficiency
ϵ	effectiveness

which gives the maximum power output of the machine. Further extension work on finite time thermodynamics related to the Rankine and Brayton heat engines was also carried out by earlier workers [2–4].

The Stirling engine has also captivated several generations of engineers and physicists due to its potential for a high conversion efficiency. However, use of this engine did not prove successful due to the relatively poor material technologies available at that time. As the world community has become much more environmentally conscious, further attention has again been paid to Stirling engines because they are inherently clean and thermally more efficient. Moreover, as a result of advances in material technology, these engines are currently being considered for a variety of applications. Stirling engines are also under research and development for their use as heat pumps, replacing conventional systems that are not ecofriendly and environmentally acceptable. Coal-fired Stirling engines have also been proposed for use in stationary power generation systems.

Currently, practical engineers make use of the ideal air standard Stirling cycle with regeneration

as a basis for analyzing trends within real Stirling engines. Badescu [5] analyzed the performance of a solar-powered heat engine operating in a Stirling cycle and studied the influence of design and climatological parameters on both the optimal solar-receiver temperature and overall thermal efficiency. Blank et al. [6] studied the power optimization of an endoreversible Stirling cycle and provided an estimate of potential performance for a real engine. Trukhow et al. [7] investigated the energy balance of an autonomous solar power plant with a Stirling engine and showed that the electric power output is proportional to the direct solar radiation. Ladas and Ibrahim [8] have presented a finite time analysis of the Stirling engine and discussed the effect of heat transfer contact time and regeneration process on power output and thermal efficiency. Wu and Blank [9] examined the power optimization of an extra-terrestrial solar radiant Stirling heat engine with an ideal/perfect regenerator and infinite heat capacity of external heat (source/sink) reservoirs. They showed that the optimal power and the corresponding thermal efficiency are based on higher and lower temperature bounds. Erbay and Yavuz [10] analyzed the real Stirling heat engine for maximum power output conditions using polytropic processes as well as regenerative losses. They also determined the efficiency and compression ratio at maximum power density and ascertained the thermal design bounds. Chen [11] investigated the effect of regenerative losses on maximum power and the corresponding thermal efficiency of an endoreversible Stirling engine with infinite heat capacity of external heat source/sink reservoirs. Chen et al. [12] studied the efficiency and engine size limits of a solar driven Stirling heat engine at maximum power output. Senft [13] studied the theoretical limitations on the performance of a Stirling engine subject to limited heat transfer, external thermal and mechanical losses.

In this paper, a general analysis of finite time thermodynamics of a Stirling heat engine has been presented with finite heat capacity of external reservoirs, regenerative losses and finite effectiveness of each of the heat exchangers [i.e. a high temperature heat exchanger (HTHEX) and a low temperature heat exchanger (LTHEX)]. We have obtained the expressions for maximum power output and computed the corresponding thermal efficiency. The effect of operating temperatures, the effectiveness of the HTHEX, LTHEX and the regenerative heat exchanger on the heat transfer (Q_H and Q_L) to and from the heat engine, the regenerative heat transfer (Q_R), the maximum power output (P_m) and the corresponding thermal efficiency of the cycle have all been studied.

2. System description

An endoreversible Stirling heat engine coupled with a heat source and heat sink reservoirs of finite heat capacity rates and with a real regenerator is depicted in Fig. 1 along with its T - s diagram in Fig. 2. This cycle approximates the compression stroke of the real heat engine as an isothermal heat rejection process (1–2) to the low temperature sink. The heat addition to the working fluid from the regenerator is modeled as the constant volume process (2–3). The expansion stroke producing work is modeled as an isothermal heat addition process (3–4) from a high temperature heat source. Finally, the heat rejection to the regenerator is modeled as the constant volume process (4–1).

As mentioned earlier, the external source/sink heat transfer processes 1–2 and 3–4 for the real Stirling heat engine must occur in a finite time, which requires that they proceed through a finite temperature difference and are, therefore, defined as being externally irreversible. During the

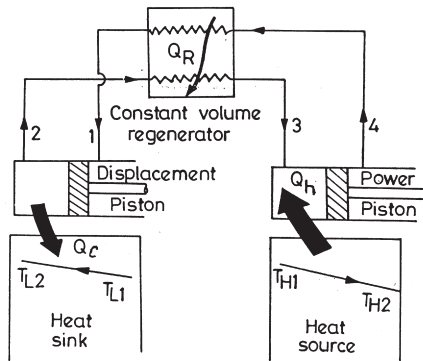
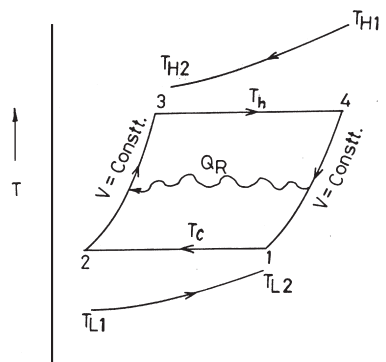


Fig. 1. Schematic diagram of the Stirling heat engine cycle.

Fig. 2. T - s diagram of the Stirling heat engine cycle.

isothermal heat rejection process 1–2, heat flows from the cycle working fluid (which is maintained at a constant temperature T_c) to the low-temperature heat sink of finite heat capacity (whose temperature increases from T_{L1} to T_{L2}). Similarly, during the isothermal heat addition process 3–4, heat is transferred from the high temperature heat source of finite heat capacity (whose temperature decreases from T_{H1} to T_{H2}) to the constant temperature (T_h) cycle working fluid. If the regenerator is an ideal one, the heat absorbed during the process 4–1 will be equal to the heat rejection during the process 2–3. However, an ideal regenerator requires an infinite time or infinite area to transfer a finite amount of heat, hence a real regenerator is also considered in the present analysis.

3. Finite time thermodynamic analysis

Fig. 2 shows the T - s diagram of an endoreversible Stirling heat engine affected by the irreversibility of finite heat transfer, Q_h and Q_c respectively, to and from the heat engine during heat addition and heat rejection processes. If V_1 and V_2 are the volumes of the working substance during the regenerative (constant volume) processes 4–1 and 2–3, respectively, the amount of heat absorbed (Q_h) and the heat released (Q_c) by the heat engine working fluid can be given as:

$$Q_h = T_h \Delta S = nRT_h \left(\ln \frac{V_1}{V_2} \right) \quad (1)$$

$$Q_c = T_c \Delta S = nRT_c \left(\ln \frac{V_1}{V_2} \right) \quad (2)$$

where n is the number of moles of the working substance and R is the universal gas constant. When the effect of finite heat transfer on the performance of thermodynamic cycles is considered, it is assumed that the heat transfer rate is proportional to the Log Mean Temperature Difference (LMTD) and written as:

$$Q_h = U_H A_H (\text{LMTD})_H t_h = C_H (T_{H1} - T_{H2}) t_h \quad (3)$$

$$Q_c = U_L A_L (\text{LMTD})_L t_c = C_L (T_{L2} - T_{L1}) t_c \quad (4)$$

where t_h and t_c are heat addition and heat rejection times for the Stirling heat engine, $U_H A_H$ and $U_L A_L$ are the overall heat transfer coefficient and area products for hot and cold sides, respectively. C_H and C_L are, respectively, the heat capacitance rates of external fluids in heat source/sink reservoirs. $(\text{LMTD})_H$ and $(\text{LMTD})_L$ are defined as:

$$(\text{LMTD})_H = \frac{[(T_{H1} - T_h) - (T_{H2} - T_h)]}{\ln \left[\frac{(T_{H1} - T_h)}{(T_{H2} - T_h)} \right]} \quad (5)$$

$$(\text{LMTD})_L = \frac{[(T_c - T_{L1}) - (T_c - T_{L2})]}{\ln \left[\frac{(T_c - T_{L1})}{(T_c - T_{L2})} \right]} \quad (6)$$

where T_h and T_c are the hot and cold side working fluid temperatures. Thus from Eqs. (3)–(6)) we get:

$$T_{H2} = T_h + (T_{H1} - T_h) e^{-N_H} \quad (7)$$

$$T_{L2} = T_c - (T_c - T_{L1}) e^{-N_L} \quad (8)$$

where N_H , N_L are the number of heat transfer units; $N_H = U_H A_H / C_H$ and $N_L = U_L A_L / C_L$. Combining Eqs. (3) and (4) with Eqs. (7) and (8) yields

$$Q_h = C_H \epsilon_H (T_{H1} - T_h) t_h \quad (9a)$$

$$Q_c = C_L \epsilon_L (T_c - T_{L1}) t_c \quad (9b)$$

where ϵ_H , ϵ_L are the effectiveness of hot and cold side heat exchangers, respectively, and are defined as:

$$\epsilon_H = 1 - \exp(-N_H) \text{ and } \epsilon_L = 1 - \exp(-N_L) \quad (10)$$

It is important to mention that there also exists a finite heat transfer in the regenerative processes besides the two isothermal heat addition and rejection processes. Thus, regenerative heat transfer (Q_R) is given by:

$$Q_R = nC_v \epsilon_R (T_h - T_c) \quad (11a)$$

where C_v is the specific heat of the working substance and ϵ_R is the effectiveness of the regenerator. Thus, the regenerative heat loss in the two regenerative processes (ΔQ per cycle) is given by:

$$\Delta Q = nC_v (1 - \epsilon_R) (T_h - T_c) \quad (11b)$$

It can be seen that $\Delta Q = 0.00$, if $\epsilon_R = 1.00$. Thus net heat absorbed (Q_H) from the heat source and heat released (Q_L) to the heat sink incorporating regenerative heat loss per cycle are, respectively, given by:

$$Q_H = Q_h + \Delta Q \quad (12)$$

$$Q_L = Q_c + \Delta Q \quad (13)$$

To achieve a more realistic case, the time of the regenerative heat transfer processes (t_3 and t_4) should also be considered in the thermodynamic analysis of a Stirling engine [6]. For this purpose, it is assumed that the temperature of the engine working fluid/substance is varying with time in the regenerative processes as given by [11]

$$\frac{dT}{dt} = \pm \alpha \quad (14)$$

where α is the proportionality constant which is independent of the temperature but dependent on the property of the regenerative material. The positive and negative signs correspond to constant volume heating and cooling processes, respectively. Using Eq. (10), one can obtain the time (t_R) of two constant volume regenerative processes as given by:

$$t_R = t_3 + t_4 = 2\alpha(T_h - T_c) \quad (15)$$

Thus the total cycle time is given by:

$$t = t_h + t_c + t_R \quad (16)$$

For the thermodynamic cycle 1–2–3–4–1, work and the power output per cycle is given by:

$$W = Q_H - Q_L = Q_h - Q_c \quad (17a)$$

$$P = \frac{W}{t} = \frac{(Q_H - Q_L)}{t} = \frac{(Q_h - Q_c)}{(t_h + t_c + t_R)} \quad (17b)$$

Using Eqs. (1, 2, 9a) and (9b), we get power output as given by:

$$P = \frac{(T_h - T_c)}{\left[\frac{T_h}{k_1(T_{H1} - T_h)} + \frac{T_c}{k_2(T_c - T_{L1})} + a_1(T_h - T_c) \right]} \quad (18)$$

and the thermal efficiency η is given by:

$$\eta = \frac{W}{Q_H} = \frac{(Q_h - Q_c)}{(Q_h + \Delta Q)} = \frac{(T_h - T_c)}{T_h + a_2(T_h - T_c)} \quad (19)$$

where $k_1 = C_H \epsilon_H$, $k_2 = C_L \epsilon_L$, $a_1 = 2/nR\alpha \ln(V_1/V_2)$ and $a_2 = C_v(1 - \epsilon_R)/R \ln(V_1/V_2)$. For the sake of convenience, let us introduce two parameters such that $x = T_h/T_c$ and $y = T_c$ in Eqs. (18) and (19) we get:

$$P = \frac{(x-1)}{\left[\frac{x}{k_1(T_{H1} - xy)} + \frac{1}{k_2(y - T_{L1})} + a_1(x-1) \right]} \quad (20)$$

and

$$\eta = \frac{(x-1)}{x + a_2(x-1)} \quad (21)$$

Eq. (20) shows that the power output (P) is a function of x and y for given parameters like T_{H1} , T_{L1} , ϵ_H , ϵ_L , C_H , C_L etc. Thus maximizing power (P) wrt x and y , we have $\frac{\partial P}{\partial x} = 0$ and $\frac{\partial P}{\partial y} = 0$ which yields optimal values of x and y as given by:

$$x = \sqrt{\frac{T_{H1}}{T_{L1}}} \text{ and } y = \frac{T_{L1} \left[1 + \sqrt{\frac{k_1 T_{H1}}{k_2 T_{L1}}} \right]}{(1 + \sqrt{k_1/k_2})}$$

Substituting the values of x and y into Eqs. (20) and (21), the maximum power output is given by

$$P_{\max} = \frac{K \left[\sqrt{T_{H1}} - \sqrt{T_{L1}} \right]^2}{\left(1 + Ka_1 \left[\sqrt{T_{H1}} - \sqrt{T_{L1}} \right]^2 \right)} \quad (22)$$

and the corresponding thermal efficiency at maximum power will be:

$$\eta_m = \frac{\left[1 - \sqrt{\frac{T_{L1}}{T_{H1}}} \right]}{\left(1 + a_2 \left[1 - \sqrt{\frac{T_{L1}}{T_{H1}}} \right] \right)} = \frac{\eta_{CA}}{(1 + a_2 \eta_{CA})} \quad (23)$$

where $K = k_1 k_2 / [\sqrt{k_1} + \sqrt{k_2}]^2$ and $[\eta_{CA} = 1 - \sqrt{(T_{L1}/T_{H1})}]$ is the Curzon–Ahlborn efficiency of an endoreversible Carnot heat engine with corresponding source/sink temperatures. It is seen from Eq. (22) that the maximum power output of the Stirling heat engine is not affected by the regenerative heat loss, although it depends upon the time of regenerative processes. Eq. (23) also clearly shows that when a Stirling heat engine with regenerative loss is operating in the maximum power output, its efficiency is different (always smaller) from that of an endoreversible Carnot heat engine efficiency (η_{CA}).

3.1. Special cases:

(i) When $\epsilon_R = 1.00$, the Stirling engine achieves the condition of perfect/ideal regeneration, although the time of regenerative process is still considered. Then the maximum power output and the corresponding thermal efficiency are given by:

$$P_{\max} = \frac{K \left[\sqrt{T_{H1}} - \sqrt{T_{L1}} \right]^2}{\left(1 + Ka_1 \left[\sqrt{T_{H1}} - \sqrt{T_{L1}} \right]^2 \right)} \quad (24)$$

and

$$\eta_m = \left[1 - \sqrt{\frac{T_{L1}}{T_{H1}}} \right] = \eta_{CA} \quad (25)$$

Thus for $\epsilon_R=1.00$, the performance of the endoreversible Stirling heat engine is with perfect/ideal regenerator in which the time of regeneration is given by Eq. (15), the optimal performance of the endoreversible Stirling heat engine is identical to that of an endoreversible Carnot heat engine. In such a case its maximum power output and the corresponding thermal efficiency are respectively given by Eqs. (24) and (25). However, physically for finite regenerative time ϵ_R should be less than unity. This shows that in the investigation of the Stirling heat engine, it would be impossible to obtain new conclusions if the regenerator losses were not considered.

(ii) When the time of regenerative processes is directly proportional to the mean time of two isothermal processes, i.e.

$$t_R = \gamma(t_h + t_c) \quad (26)$$

where γ is the proportionality constant then

$$P_{\max} = \frac{K \left[\sqrt{T_{H1}} - \sqrt{T_{L1}} \right]^2}{1 + \gamma} \quad (27)$$

(iii) When the regenerative time is zero, i.e. $a_1=0.00$, the maximum power output is given by

$$P_{\max} = K \left[\sqrt{T_{H1}} - \sqrt{T_{L1}} \right]^2 \quad (28)$$

and the corresponding thermal efficiency in both cases is still given by Eq. (23).

Thus it is seen from Eqs. (27) and (28) that the maximum power output of a Stirling heat engine is equivalent to that of an endoreversible Carnot heat engine for the same operating conditions in which the time of regeneration process is given by Eqs. (15) and (26) but the thermal efficiency will be lower.

(iv) Also, in the case when $\epsilon_H=\epsilon_L=1.00$ and $\epsilon_R<1.00$, i.e. the heat source and heat sink are of infinite heat capacitance, i.e. the temperature of heat source and heat sink are such that $T_{H1}=T_{H2}=T_H$ and $T_{L1}=T_{L2}=T_L$ and t_R is given by Eq. (26), the power output becomes:

$$P_{\max} = \frac{K \left[\sqrt{T_H} - \sqrt{T_L} \right]^2}{1 + \gamma} \quad (29)$$

and the corresponding thermal efficiency is still given by Eq. (23) along with $\eta_{CA}=1-\sqrt{T_L/T_H}$.

4. Numerical results and discussion

In order to have a numerical appreciation of results, we have considered the inlet source and sink temperatures as $T_{H1}=1300$ K and $T_{L1}=300$ K, the volume ratio as $V_1/V_2=2.5$, the effectiveness of each heat exchanger, i.e. the hot/cold side heat exchanger and the regenerator in the range from 0.40 to 1.00, and the heat capacitance rates of the heat source/sink reservoirs in the range from 0.30 to 1.80 kW/K. We have studied the effect of each of these parameters (while keeping others constant) on the heat transfers (Q_H and Q_L) to and from the heat engine, the regenerative heat transfer (Q_R), the output work (W), the maximum power output (P_m) and the corresponding thermal efficiency of the Stirling heat engine. The discussion of results is given below.

Tables 1(a–c) show the effects of the effectivenesses (ϵ_H , ϵ_L and ϵ_R) of various heat exchangers of the hot side, the cold side and the regenerator, respectively.

4.1. Effect of ϵ_H

It is seen from Table 1(a) that as the effectiveness of the hot side heat exchanger increases, the heat transfers (Q_H and Q_L), the regenerative heat transfer (Q_R), the output work (W) and the maximum power output (P_m) increase while the thermal efficiency remains constant. The effect of ϵ_H is more pronounced for the maximum power output (P_m) and less pronounced for the heat input (Q_H) to the heat engine.

4.2. Effect of ϵ_L

It is seen from Table 1(b) that as the effectiveness of the cold side heat exchanger (ϵ_L) increases, the heat transfers (Q_H and Q_L), the regenerative heat transfer (Q_R) and the output work (W) decrease but the maximum power output (P_m) increases, while the thermal efficiency remains constant. The effect of ϵ_L is more pronounced for the maximum power output (P_m) and less pronounced for the heat transfer (Q_L) from the heat engine.

4.3. Effect of ϵ_R

It is seen from Table 1(c) that as the regenerator effectiveness (ϵ_R) increases, the heat transfers (Q_H and Q_L) decrease but the regenerative heat transfer (Q_R) and the thermal efficiency (η_m) increase while the output work (W) and the maximum power output (P_m) remain constant. The effect of ϵ_R is more pronounced for the heat transfer from the heat engine (Q_L) and less pronounced for the thermal efficiency (η_m). It is also seen that when there is no heat loss through the regenerator ($\epsilon_R=1.00$), the Stirling heat engine attains efficiency equal to the Curzon–Ahlborn efficiency (η_{CA}) of an endoreversible Carnot heat engine. As is seen from the table, the regenerator effectiveness should be high enough from the point of view of higher thermal efficiency as well as for lower internal irreversibility in the heat engine. However, an endoreversible Stirling heat engine with ideal regenerator is as efficient as an endoreversible Carnot heat engine but it is not practical because an ideal regeneration requires infinite time or infinite regenerative area.

Tables 2(a–b) shows the effects of external heat source/sink reservoirs inlet fluid temperatures (T_{H1} and T_{L1}), on various parameters, i.e. the heat transfers (Q_H and Q_L) to and from the heat

Table 1
Effect of ϵ_H , ϵ_L and ϵ_R on heat transfer, power output and thermal efficiency ($\epsilon_R=0.90$, $\epsilon_H=\epsilon_L=0.80$, $\eta_m=43.93\%$, $T_{H1}=1300\text{K}$, $T_{L1}=300\text{K}$ and $C_H=C_L=1.00\text{ kW/K}$)

ϵ_H or ϵ_L or ϵ_R	(a) Effect of ϵ_H					(b) Effect of ϵ_L					(c) Effect of ϵ_R				
	Q_H (kJ)	Q_L (kJ)	Q_R (kJ)	W (kJ)	P_m (kW)	Q_H (kJ)	Q_L (kJ)	Q_R (kJ)	W (kJ)	P_m (kW)	Q_H (kJ)	Q_L (kJ)	Q_R (kJ)	η_m (%)	
0.40	212.79	119.31	296.03	93.48	48.16	240.06	134.60	333.97	105.46	48.16	401.42	301.96	140.00	24.78	
0.50	217.13	121.74	302.07	95.38	54.71	235.72	132.17	327.93	103.55	54.71	366.42	266.96	175.00	27.15	
0.60	220.72	123.76	307.06	96.96	60.44	232.13	130.16	322.94	101.97	60.44	331.42	231.96	210.00	30.01	
0.70	223.77	125.47	311.31	98.30	65.54	229.08	128.44	318.69	100.63	65.54	296.42	196.96	245.00	33.56	
0.80	226.42	126.96	315.00	99.47	70.13	226.42	126.96	315.00	99.47	70.13	261.42	161.96	280.00	38.05	
0.90	228.76	128.27	318.25	100.49	74.32	224.08	125.65	311.75	98.44	74.32	226.42	126.96	315.00	43.93	
1.00	230.85	129.44	321.16	101.41	78.15	221.99	124.47	308.84	97.52	78.15	191.42	91.96	350.00	51.96	

Table 2
Effect of T_{H1} and T_{L1} on heat transfer, power output and thermal efficiency ($\epsilon_R=0.90$, $\epsilon_H=\epsilon_L=0.80$, $T_{L1}=300\text{K}$ and $C_H=C_L=1.00\text{ kW/K}$)

(a) Effect of T_{H1}							(b) Effect of T_{L1}						
T_{H1} (K)	Q_H (kJ)	Q_L (kJ)	Q_R (kJ)	W (kJ)	P_m (kW)	η_m (%)	T_{L1} (K)	Q_H (kJ)	Q_L (kJ)	Q_R (kJ)	W (kJ)	P_m (kW)	η_m (%)
900	162.20	102.52	189.00	59.68	32.14	36.79	290	225.73	125.27	318.15	100.46	72.33	44.51
1000	178.45	108.82	220.50	69.63	40.89	39.02	295	226.08	126.11	316.57	99.96	71.22	44.22
1100	194.55	114.98	252.00	79.57	50.18	40.90	300	226.42	126.96	315.00	99.47	70.13	43.93
1200	210.54	121.02	283.50	89.52	59.95	42.52	305	226.76	127.79	313.42	98.97	69.06	43.64
1300	226.42	126.96	315.00	99.47	70.13	43.93	310	227.10	128.63	311.85	98.47	68.01	43.36
1400	242.21	132.80	346.50	109.41	80.68	45.17	315	227.43	129.46	310.27	97.97	66.97	43.08
1500	257.92	138.56	378.00	119.36	91.56	46.28	320	227.76	130.28	308.70	97.48	65.95	42.80

engine, the regenerative heat transfer (Q_R), the output work (W), the maximum power output (P_m) and the thermal efficiency (η_m) of the stirling heat engine.

4.4. Effect of T_{H1}

It can be seen from Table 2(a) that as the external heat source fluid inlet temperature (T_{H1}) increases, the heat transfers (Q_H and Q_L), the regenerative heat transfer (Q_R), the output work (W), the maximum power output (P_m) and the thermal efficiency (η_m) all increase. The effect of T_{H1} is more pronounced for the maximum power output (P_m) and less pronounced for the thermal efficiency (η_m). Thus it is desirable to have a high temperature heat source from the point of view of higher power output as well as higher thermal efficiency.

4.5. Effect of T_{L1}

It is seen from Table 2(b) that as the heat sink working fluid inlet temperature (T_{L1}) increases, the heat transfers (Q_H and Q_L) increase, while the regenerative heat transfer (Q_R), the output work (W), the maximum power output (P_m) and the thermal efficiency (η_m) decrease. The effect of T_{L1} is more pronounced for the maximum power output (P_m) and less pronounced for the heat input (Q_H) to the heat engine. Thus it is desirable to have a low temperature heat sink from the point of view of higher power output (P_m) as well as higher thermal efficiency (η_m).

Tables 3(a–b) show the effects of the heat capacitance rates (C_H and C_L) of the source/sink

Table 3

Effect of C_H and C_L on heat transfer, power output and thermal efficiency (assuming $\epsilon_R=0.90$, $\epsilon_H=\epsilon_L=0.80$, $T_{H1}=1300\text{K}$, $T_{L1}=300\text{K}$, $C_L=1.00\text{ kW/K}$ and $\eta_m=43.93\%$)

C_H or C_L (kW/K)	(a) Effect of C_H				(b) Effect of C_L				
	Q_H (kJ)	Q_L (kJ)	Q_R (kJ)	W (kJ)	Q_H (kJ)	Q_L (kJ)	Q_R (kJ)	W (kJ)	P_m (kW)
0.30	203.20	113.93	282.69	89.26	249.65	139.98	347.31	109.67	35.16
0.40	208.53	116.92	290.11	91.61	244.32	136.99	339.89	107.53	42.13
0.50	212.79	119.31	296.03	93.48	240.06	134.60	333.67	105.46	48.16
0.60	216.33	121.30	300.96	95.03	236.52	132.62	329.04	103.90	53.47
0.70	219.36	122.99	305.17	96.36	233.49	130.92	324.83	102.57	58.23
0.80	221.99	124.47	308.84	97.52	230.85	129.44	321.16	101.41	62.55
0.90	224.33	125.78	312.09	98.55	228.52	128.13	317.91	100.39	66.50
1.00	226.42	126.96	315.00	99.47	226.42	126.96	315.00	99.47	70.13
1.10	228.32	128.02	317.63	100.30	224.53	125.90	312.37	98.63	73.51
1.20	230.04	128.99	320.04	101.06	222.80	124.93	309.96	97.88	76.65
1.30	231.63	129.88	322.24	101.75	221.22	124.04	307.76	97.18	79.60
1.40	233.09	130.70	324.28	102.40	219.75	123.22	305.72	96.54	82.37
1.50	234.45	131.46	326.17	102.99	218.39	122.46	303.83	95.94	84.98
1.60	235.72	132.17	327.93	103.55	217.13	121.74	302.07	95.38	87.45
1.70	236.90	132.83	329.58	104.07	215.94	121.08	300.42	94.86	89.80
1.80	238.02	133.46	331.13	104.56	214.83	120.46	298.87	94.37	92.03

reservoirs on the heat transfers (Q_H and Q_L), the regenerative heat transfer (Q_R), the output work (W), the maximum power output (P_m) and the thermal efficiency (η_m).

4.6. Effect of C_H

It is seen from Table 3(a) that as the heat capacitance rate (C_H) of the heat source reservoir increases, the heat transfers (Q_H and Q_L), the regenerative heat transfer (Q_R), the output work (W), and the maximum power output (P_m) increase while the thermal efficiency remains constant. The effect of C_H is more pronounced for the maximum power output (P_m) and less pronounced for the regenerative heat transfer (Q_R).

4.7. Effect of C_L

Table 3(b) shows that as the heat capacitance rate of the heat sink (C_L) reservoir increases, the heat transfers (Q_H and Q_L), the regenerative heat transfer (Q_R) and the output work (W) decrease while the maximum power output (P_m) increases, but the thermal efficiency remains constant. The effect of C_L is more pronounced for the maximum power (P_m) output and less pronounced for the regenerative heat transfer (Q_R).

It can be further seen from Tables 3(a–b) that by increasing C_H , both Q_H and Q_L increase while by increasing C_L , both Q_H and Q_L decrease thereby keeping the power output always the same in both cases. This shows that by increasing C_L we have to supply less heat to the engine to gain the same power output as one would gain it with more heat input by increasing C_H . Therefore, it is desirable to have higher C_L rather than C_H from the point of view of lower heat transfers to and from the heat engine as well as higher power output. Alternatively, this can also be explained in terms of irreversibilities produced on source/sink sides of the engine. As C_H increases total irreversibilities increase while as C_L increases total irreversibilities decrease. However, internal irreversibility due to regenerative heat losses is not affected by C_H or C_L . External irreversibility on the source side decreases while on the sink side it increases as C_H increases. On the other hand, external irreversibility on the source side increases while on the sink side it decreases as C_L increases. Thus, it is desirable to have higher C_L and lower C_H rather than lower C_L and higher C_H .

5. Conclusions

Finite time thermodynamics has been applied to maximize the power output and the corresponding thermal efficiency of an endoreversible Stirling heat engine with finite heat capacitance rates of external fluids in the heat source/sink reservoirs and with regenerative losses. It is found that the effectiveness (ϵ_H and ϵ_L) of the hot and cold side heat exchangers affect only the maximum power output (P_m), while the regenerator effectiveness (ϵ_R) affects the thermal efficiency only but the external heat source/sink reservoirs fluids inlet temperatures (T_{H1} and T_{L1}) affect both the maximum power output (P_m) and the corresponding thermal efficiency. It is also desirable to have a high temperature heat source and low temperature heat sink from the point of view of higher power output (P_m) and the corresponding higher thermal efficiency. The heat capacitance rates

(C_H and C_L) of the external fluids in heat source/sink reservoirs play an important role for higher power output (P_m) as well as for lower heat input (Q_H). Increasing the heat capacitance rate (C_H) of the external fluid in the heat source reservoir means increasing the heat input (Q_H) to the heat engine for the same power output. As we know, increasing C_L means decreasing the external irreversibility between the engine and the heat sink because the higher value of C_L forces the working fluid on the sink side to reject more heat at lower temperatures. It is also found that increasing the heat capacitance rate (C_L) of the heat sink reservoir (in comparison to C_H) can reduce the heat input (Q_H) to the heat engine for the corresponding same power output. Hence, it is desirable to have larger heat capacity of the heat sink in comparison to the heat source reservoir for higher maximum power output (P_m) and lower heat input (Q_H). Thus the present analysis provides a new theoretical basis for the design, performance evaluation and improvement of a theoretical Stirling heat engine.

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